

Some Indigenous Strategies in Mathematics Teaching: Taking the Artefacts into the Classroom

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ABSTRACT This paper explores the use of indigenous materials to promote better communication and understanding of mathematical concepts among learners in classroom teaching. It is illustrated on how to use local materials to construct figures that results into proof of mathematical theorems and also examples given are illustrated using Sketchpad program to explore how this technology tool can be used to renovate traditional artefacts in teaching and learning mathematics. The authors argue that such an approach, of using local materials to teach mathematics content by relating what is taught in the classroom to the environment where the learners live, can be an effective way of communication in mathematics teaching. It is recommended that the curriculum development process include indigenous materials in the school syllabus for better understanding of mathematics content taught in class.

INTRODUCTION

The use of concrete materials in mathematics teaching dates many years back with the publications by Dienes (1960) and Bruner (1961) of the theories on the use of these materials in teaching. A number of studies on the effectiveness of using concrete materials have been conducted since Dienes' and Bruner's theories, and the views of mathematics educators have been mixed. Fennema (1972) argued that the use of concrete materials in teaching mathematics was better suited for beginning learners such as those in lower primary schools and that it could not be most useful to older students such as those in upper primary and secondary schools. Fennema's (1972) argument was contradicted by Suydam and Higgins (1977) who reported a pattern of beneficial results for all learners in primary and secondary schools. While Labinowicz (1985), Resnick and Omanson (1987) and Thompson (1992) reported that using base-ten blocks had little effect on upper-primary students' understanding of addition and subtraction algorithms, Fuson and Briars (1990) had earlier reported success in the use of base-ten

blocks in teaching addition and subtraction algorithms in these levels. Hiebert and Wearne (1991) and Wearne and Hiebert (1998) reported consistent success in the use of concrete materials to aid students' understanding of decimal fractions and decimal numeration at both upper primary and secondary school levels. As Labinowicz (1985) argued, though an actual wooden base-ten cube was more concrete to students than is a picture of a wooden base-ten cube, for students who were still constructing concepts of numeration, a wooden base-ten cube would not be more meaningful than a pictured cube. Also as argued by Thompson (1994), concrete materials do not automatically carry mathematical meaning for students if the concrete was not differentiated from the picture for meaningful understanding of intended concepts of the material taught.

The above literature however did not specify whether the concrete materials used in teaching were familiar to the learners, that is, derived from the environment familiar to the learners or whether the learners had interacted with the materials before. For students who encounter the concrete materials for the first time in class, might have a problem of understanding what the materials represent. In such a situation, students may have first to learn the operation of the materials and then to what they were intended for in the class. Such a process might not be easy for the learner to grasp intended mathematical concepts. The utilization of the learner's environment has been regarded to be advanta-

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geous to the learner to reflect on the cultural experiences and learning in the classroom (Ezeife 2003; Furuta et al 2015; Madden 2015; Volfová 2015). In the local environment there exists indigenous concrete materials familiar to the learners that can be used in the classrooms for teaching mathematics. These indigenous materials contain 'frozen' knowledge that remain unutilized in mathematics teaching (Gay and Cole 1967; Gerdes 1997; Kaino 2013).

Studies in the last decade have shown the significance of integration of indigenous knowledge in classroom instruction in teaching and learning of mathematics (Asher 2002; Eglash 2002; Kaino 2013). For example, studies done in New Zealand that examined mathematics achievement and the professional development of mathematics teachers found the importance of indigenous knowledge in the development of a national program (Aspin 1994; Smith 1999 in Sandoval 2007; Rogers 2003). Indigenous knowledge is not used or acknowledged in mathematics curricula used in most countries and a few studies like those done in Alaska show the strides made in the development of curriculum materials. In Alaska, the development of curriculum materials utilized Alaska Native constructs such as fish rack construction, egg gathering, salmon harvesting, and star navigation as an avenue for teaching mathematical content that prepared students to meet national and state standards and related assessment mandates (Adams and Lipka 2003). In the study of mathematical learning difficulties of the Kpelle people (in Liberia), Gay and Cole (1967) found that the mathematical content taught in class did not make any sense to the Kpelle-culture and the methods used were primarily based on rote memory. The study showed that Kpelle illiterate adults performed better than North American adults when solving problems like the estimation of number of cups of rice in a container. Studies in Australia have found it vital to include indigenous perspectives within school practices to ensure improved outcomes for indigenous peoples in the country (University of Sydney 2012).

The current use of technology in instruction is widely an adopted practice in teaching and learning mathematics in many countries. Many educators agree that technology supports students' conceptual understanding of real world experiences (Howard 2015). Computer programs like Sketchpad illustrated in this paper have been

used in teaching of various topics in mathematics. Some evidences have indicated that when using Sketchpad, students make sense of the words they hear (Stidham 2015) and see things they did not see previously (Brown 2015) indicating the significance of applying such technology in teaching mathematics.

The above literature indicates that when indigenous materials are used appropriately in teaching and with an aid of technology, they can make students understand better mathematical concepts taught in the class.

Objectives of the Study

The aim of the study is to demonstrate how local materials can be used (also with the aid of technology) to teach mathematics topics from the syllabus. Specifically, the illustrated activities aim to show how local materials are used to (i) form parallelograms using sticks, (ii) form a rectangle, a square and prove theorems of these, (iii) determine different base areas of squares, rectangles and circles using strings, and (iv) prove why circular base areas are popular in construction of houses in the local environment.

The Construction of Rectangular and Square Houses

Background knowledge required by the students to understand and use the materials illustrated

Grade 6. Classification of quadrilaterals-square, rectangle, trapezium and kite; similar and congruent figures.

Grade 7. Classification of quadrilaterals-square, rectangle, rhombus and quadrilateral; similar and congruent figures.

Grade 8. Classification of quadrilaterals-rectangle, rhombus and trapezoids, kite; similar and congruent figures.

Grade 9. Classification of quadrilaterals-figures of sides more than 4 sides.

Grade 10. Identification of congruent figures.

Materials Needed in the Lesson

The materials that are used by local people to construct rectangles houses are brought into the classroom for demonstration.

Classroom Activity 1

In the classroom, students start by laying down on the floor two long sticks of equal lengths. Then these first two sticks are combined with two other sticks also of equal length, but shorter than the first ones. And then form a quadrilateral as shown in Figure 1.

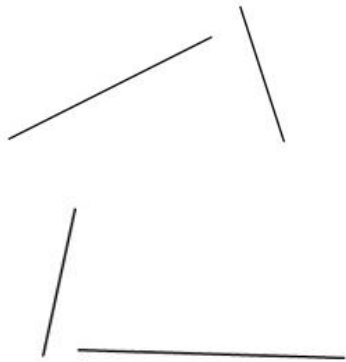


Fig. 1. Laying of four sticks to form a quadrilateral

Now the students move manually the sticks to form a quadrilateral (Fig. 2).



Fig. 2. Formation of a parallelogram

Students continue adjusting the figure to make the diagonals equal. The diagonals have to be physically measured. After the diagonals are found equal, the lines are drawn on the floor to form the figure which is a rectangle (Fig. 3).



Fig. 3. Formation of a rectangle

Students can now analyze the properties of the rectangle: that the diagonals are equal and bisect at the centre of the rectangle. In addition, the opposite lengths are equal and parallel. These properties prove that the quadrilateral is a rectangle (proof that an equi-diagonal parallelogram is a rectangle).

Repeating the same procedure as in Figure 1 up to the procedure in Figure 3 using the sticks with the same length, another quadrilateral in Figure 4 is formed. Students can now analyze the figure by measuring physically: that (i) the diagonals are equal and (ii) the diagonals bisect each other, and (iii) the diagonals bisect at right angles. Students will then have proved the theorem that the diagonals of the square bisect at right angles.

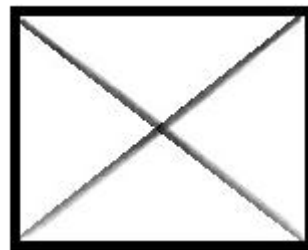


Fig. 4. Formation of a square

With Figures 3 and 4, students will have demonstrated how the rectangular and square bases of local houses are set. With these bases, the erections of the rectangular and square buildings start.

Illustration by Sketchpad Program

Stage 1: Create a Parallelogram

Students are guided to create a parallelogram with sketchpad.

1. Starting from a point, they will draw two sides and mark one side as a vector.
2. Select another side and translate it with the vector marked.
3. Repeat step 2 for another side to complete the parallelogram.
4. Drag any vertex to see the result.

Here the translation is to obtain an opposite side which is parallel and of equal length to the

original side. Students are able to determine various lengths of the opposite sides of a parallelogram and obtain the sizes of the opposite angles to learn more about the parallelogram. This process is obtained by dragging the vertices in different ways to measure the lengths and sizes of angles.

Stage 2: Testing the Lengths of the Diagonals of a Parallelogram

A sketchpad is designed where there is a parallelogram that all sides of it have the fixed lengths. This is done with two circles centred at the first vertex (Fig. 5). The second and third vertices lay on two circles respectively. Using translation, the parallelogram is completed and the circles are hidden.

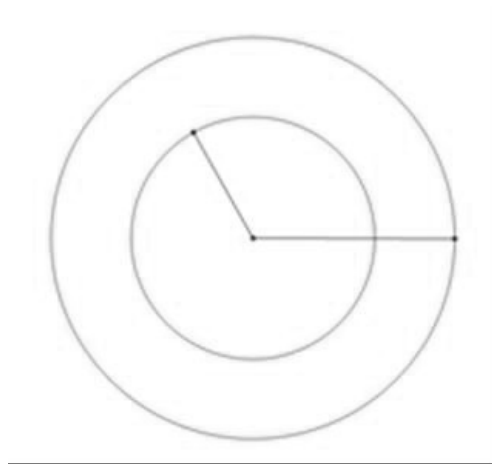


Fig. 5. Sketch process of the construction of the parallelogram

Students are guided on how to measure the lengths of diagonals first, that is to select the line segment, and choose length from the measure dropdown menu. Students adjust the shape of the parallelogram and observe the lengths of diagonals until two diagonals are equal in length. Students then determine the angle at one corner of the parallelogram and find out that the parallelogram now turns to a rectangle. Students further investigate on the areas of shapes, as its lengths of diagonals change. Through investigations it would be realized that upon translation, the area of the parallelogram would remain the same as long as the base and heights do not change.

Comparison of Rectangular, Square and Circular Bases

Background Knowledge Required by the Students

Grade 6. Areas of squares and rectangles; areas of parallelograms and trapezoids; parts of a circle: centre of a circle.

Grade 7. Areas of squares and rectangles; perimeter of squares and rectangles; areas of parallelograms and trapezoids; parts of a circle: centre of a circle.

Grade 8. Areas of parallelograms and trapezoids; parts of a circle- radius, chord, diameter; circles-word problems.

Grade 9. Areas and perimeter of similar figures.

Materials Needed in the Lesson

Rulers, threads, pencils, compasses and calculators

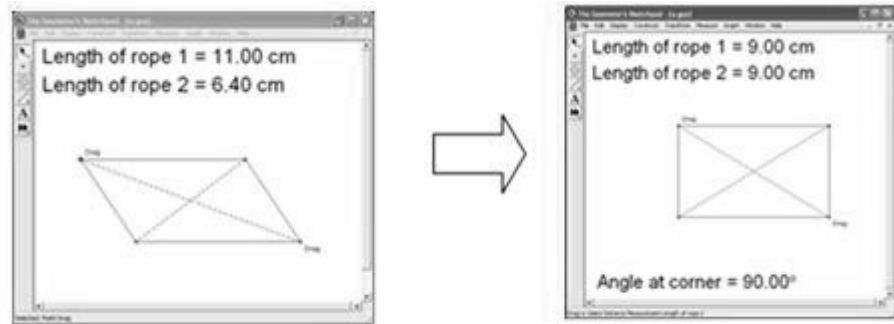


Fig. 6. Sketchpad transformation of the parallelogram into a rectangle

Classroom Activity 2

The threads/strings of any length are cut and students can construct figures of circular, rectangular and circular shapes using these threads. These figures can easily be constructed using ruler, pencil and compasses than fixing threads on the ground. Choosing simple dimensions of figures would make constructions easy. In this activity students choose a certain length to construct the three figures and this serves as a *perimeter* for all constructed figures (Fig. 7). Students can be guided to construct figures with different perimeters. For example, a circle with a diameter of 14 cm has a circumference of about 44 cm. A square of equivalent perimeter measures 11 cm on the side; a 7-by-15 cm rectangle also has a perimeter of 44 cm. This length (of the perimeter) is used as a model of the actual length used in the construction of local house bases. Now students compute the areas of the three figures with the same perimeter of 44 cm.

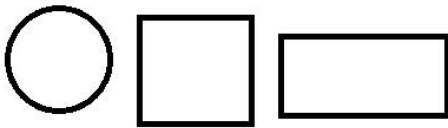


Fig. 7. A circle, square and rectangle of same perimeter value

Area of the circle = $22/7 \times 7^2 = 154 \text{ cm}^2$

Area of the square = $11 \times 11 = 121 \text{ cm}^2$

Area of the rectangle = $7 \times 15 = 105$

From above calculations, for a given perimeter of 44 cm, the circle provides a bigger area (of 154) than the square and the rectangle with areas 121 and 105 respectively. The area of the square is smaller than the area of the circle by 33, or 21 percent. With the rectangle, it is by 49 or 32 percent smaller than the circle.

Illustration with Sketchpad

Maximum Area of Shapes: In this investigation, students examine the area of a shape with a fixed perimeter, and find the shape of maximum area. Before the activity, students are assumed to be familiar with some properties of regular polygons and circles.

Conjecture

Students open the sketch designed by the teacher. On top of the sketch there are five poly-

gons. Their perimeters and areas are listed in the table below.

1. Drag the vertex of the polygon so that all five polygons have the same perimeter.
2. Examine their areas. What trend of the areas can you notice when the number of sides increases?
3. Drag the point on the circle so that it has circumference same as the perimeter of the polygons.
4. Observe the area of the circle. What features make you sure that the circle is the shape with the maximum area?
5. There is an irregular polygon in the sketch where all vertexes can be dragged. You can explore different shapes to justify your conjecture.

The exercise shows how students can be able to construct a variety of shapes and determine their areas easily by using different circumference lengths indicating the circle to have the maximum area.

Summary

The use of sketchpad in the two activities above shows the advantage of this facility over the traditional ways in geometric problem solving. This technology shows that:

1. More activities that explore deeper concepts can be manipulated in a desktop environment.
2. Materials are readily available on the desktop rather than being of physical nature (that involve collection of materials, that is, threads, cutting them to required sizes, fixing, manually stretching to get exact shapes, etc.).
3. Student investigations produce rapid results in a more efficient way.
4. Sketchpad has many powerful features, such as; Measuring, Adding controller, Create a shape with certain conditions, etc., which are user friendly and useful in the classroom activity design.
5. Students are given the opportunity to discover widely and compile mathematical explorations from their experiments.

Sketchpad process has a conceptual dimension of artefacts that make descriptions in diagrams and contain a concrete approach of knowledge abstraction from the artefact before the theory or material is necessarily introduced to the learner. The process illustrated indicates how

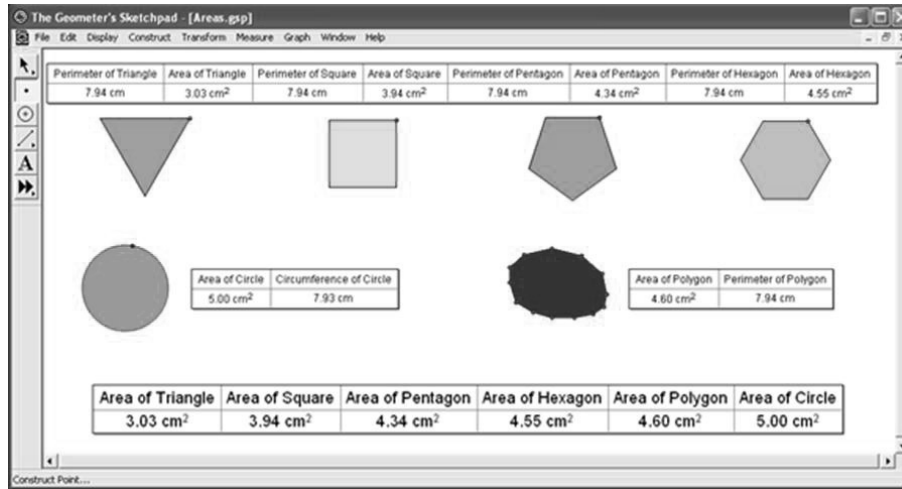


Fig. 8. Sketchpad determination of base areas

the learner is helped to construct objects and then explore their mathematical properties by using the tools available. Examples provided encourage the process of discovery in which students visualize and analyze the problem even before attempting a proof. This way of instruction tends to improve conceptualization and mathematical problem solving skills among students with a quick and friendly knowledge delivery process.

DISCUSSION

The illustrations in Figures 2 to 6 involve classification of quadrilaterals (rectangle, rhombus, trapezoids and the kite). These shapes are taught to students in Grades 6-9 in the South African mathematics syllabus. The materials illustrated gear the students to construct practical ideas on parallelograms from which they obtain rectangles and squares. At this stage, students get ideas on the proofs of theorems involving a rectangle and a square. Formal proofs of the theorems are done in Grades 11 and 12 and pre knowledge of the construction of rectangles and squares at lower grades would be advantageous in the proofs of the theorems at higher grades. Figure 7 illustrates square, rectangular and circular bases of the same perimeter, the knowledge students are expected to get in Grades 6-9. In these levels, students learn to determine areas of squares, rectangles, parallel-

ograms, trapezoids and a circle. From the concrete objects they construct in class, students would be able to determine different areas of squares, rectangles and circles for comparison. In activity 1, students prove that for a given length, inscribing the three figures, that is, a circle, square and rectangle, the circle has the maximum base area. From these findings, students will have found the reason why circular base houses are popular in the local environment. In addition, students will have realized that only looking at the constructed figures could not tell the largest area until the calculations were practically determined. One of the reasons of the popularity of circular base houses in the local environment is the advantage of occupying more base area than other shapes. Furthermore, it was found that round houses have the lowest requirements in terms of materials and expenditure of time and energy (Zaslavsky 1990).

The competency of the students in construction of illustrated figures and proof of theorems in class using indigenous materials can be called concepts-in-action and theorems-in-action (Vergnaud 1998). The examples illustrated show that if students were engaged in such activities from Grades 6 to 10, the understanding of concepts taught in the construction of parallelograms, rectangles and circles leading also to the proofs of the theorems at higher grades in Grades 11 and 12 could be easily grasped by the students in mathematics learning. The use of mate-

rials as illustrated in the activities shall be meaningful if they reflect the materials known to the learners as well as related to the mathematical content studied. As Thompson (1994) argued, concrete materials would not automatically carry any mathematical meaning for students if they were not used to realize stipulated objectives. Studies in the last decade (Asher 2002; Eglash 2002; Furuta et al. 2015) support the integration of indigenous knowledge in classroom instruction in teaching and learning of mathematics and other subjects. The inclusion of indigenous knowledge in the school curricula is considered significant in the improvement of student outcomes (University of Sydney 2012; Madden 2015; Volfová 2015; Kaino in press). And the illustrations using Sketpad have shown how local available materials can be utilized to deliver knowledge more effectively as also shown by Watson (2015) on the increase of student motivation and performance when technology is integrated into school mathematics instruction (Fig. 8).

CONCLUSION

In this paper, it has been argued that local materials can be used to teach mathematics topics from the syllabus and in this case referred from the South African mathematics syllabus. The activities show that students can learn mathematics content through the indigenous approach of using materials available in the local environment by reflecting on the content taught and also by the aid of technology. The proof of the theorems using illustrated materials indicated the significance of using such materials at the lower levels that would give advantage to students at higher levels to grasp easily the knowledge taught when students reflect on what they previously learnt. The materials used in class should always carry the mathematical meaning to achieve stipulated objectives of the lesson. The use of sketchpad has demonstrated that learners are provided with an additional opportunity to discover quickly and learn more widely. The use of indigenous materials for competency in teaching and learning mathematics would be regarded as an effective way of communication for easy grasping of mathematical knowledge.

RECOMMENDATIONS

It is recommended, as a result of this study, that the integration of indigenous materials and technology in the school mathematics curriculum should be part of the curriculum development process for better understanding of mathematical concepts..

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